

How to make decisions with risk?

- We live in a stochastic world. This makes choosing the best option nontrivial.
- We can quantify this with random variables X and their associated distributions

$$\text{Prob}(a < X < b)$$

$$= \int_a^b p(x) dx$$



Applications

- Finance** – which investment to choose, and how to balance portfolios
- Biology** – Reproductive resource allocation among offspring
- Research** – which projects to work on or fund with grants
- Social insurance** – altruism towards unlucky individuals



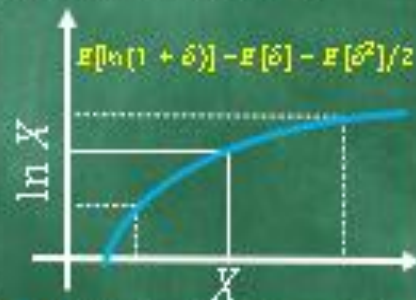
–Vs–



What is the right number to assign a distribution?

- $E[X]$ 1600's – Pascal, Fermat, Bernoulli(s)
- $E[\ln X]$ 1956 – Kelly, $P_i = \prod X \rightarrow \ln P_i = \sum \ln X$
Justified by the Central Limit Theorem

X	$E[X]$	$E[\ln X]$
8, 8	8	3
1, 16	8.5	2



- Max $E[X]$ given $\text{Var}[X] < k$ 1952 – Markowitz
 $\sim E[X] - \lambda \text{Var}[X]$

Bet Hedging is a mean/variance tradeoff by λ

Covariance

$$X = \sum w_i X_i \rightarrow E[X] = w^T \vec{E} \quad \text{Var}[X] = w^T \Sigma w$$

- $E[X] - \frac{\text{Var}[X]}{N}$ 1974 – Gillespie,
Let mean and variance coevolve

Classified Bet Hedging into:

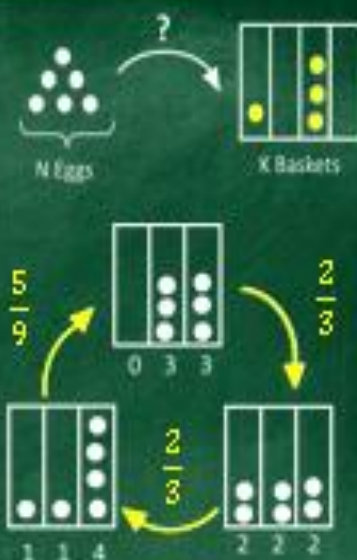
- Conservative** – minimize var
- Diversified** – minimize correlation
- Adaptive** – to reliable environmental cues



Could there be no "best"?

1970 - Efron

- Each player rolls a fair die, the higher number wins
- How to distribute eggs (dots) across baskets (faces)?



X is preferred to Y if
 $P(X > Y) > \frac{1}{2}$

Later Research / Open Questions

- Debates over empirical evidence for bet hedging
- Time varying or uncertain distributions
- Descriptive approaches in humans
- Effects of significant tail events / extinction risk, and finite time horizons

Notes and references on my website!

to make decisions with risk?

due to **non determinism** from quantum mechanics or **imperfect information**, the future state of the system is uncertain. This makes decisions nontrivial, as any action can have a **range of consequences**.

Many of answers to this question.

Applications

Many examples of the same abstract problem: how to **distribute resources** (money or effort) among uncertain endeavors (investments, offspring, research projects).

Each distribution creates a random variable – the portfolio performance.

What is the right number to assign a distribution?

- We know how to **compare numbers**, so it would suffice to **compress** the distribution.
- Actuarial / insurance: CLT - **temporal** through sequential bets or **spatial** through concurrent, either must be **independent**
- Problem: resource often grow **multiplicatively**
- Kelly (bell labs **information theory**) solved for the bet proportion that **maximizes growth rate = information rate**.
- The explanation for this difference is the **variance** an average (squared) deviation from $E[X]$.
- **Geometric intuition**: variance is symmetric by log is not, +1 vs +1%.
- Variance is bad, how to **co-optimize** with $E[X]$?
- Markowitz (**economist**) **Computed the efficient** set of RV. Tradeoff **locally looks linear** by LaGrange multipliers
 - This “Modern portfolio theory” won **1990 economics Nobel prize**.
 - **Doesn't give a resolution**, like saying which vector is longer, there is a range of answers based on tradeoff.
- Gillespie (**biologist**) **pins down a lambda**.
- Phenotypic variance (**diversity**) reduces aggregate fitness **variance**.

Could there be no “best”?

- What if fitness is **relative** not absolute?
- Efron (**statistician**) concrete formulation
- Bio **low survival** then harsh **competition**
- If it was **transitive**, 033 should beat 114
- But this only happens **4/9** of the time.
- We have a **rock-papers-scissors** dynamic
- Also works for **multiplayer**, any **digraph**.
- Might object to payoff, $[2,2,2] > [1,1,9999999]$, but doesn't need **strict threshold** or **independent rolls**

Later Research / Open Questions

- **Diversified BH** vs **plasticity/parental effects**, **microbes** experiments fast evolution environment control.
- We assumed the only **source of uncertainty** was the inherent stochasticity, and no uncertainty in the distribution. It is **costly to reduce uncertainty** in the distribution e.g. in clinical trials. This creates an **Explore-exploit** tradeoff, investigated by **learning** theories.
- **Prospect theory** explains biases (risk aversion or seeking). Kahneman received the **2002 Nobel Prize** in economics for this. Also **Allais paradox**.

Subsequent Research and open questions

- **Debates over empirical evidence for bet hedging** – hard to distinguish from ordinary plasticity, constraint, parental effects, or unmeasured adaptation. Examples in desert annuals and then microbes for fast evolution / exact control of environment.
- **What if we don't have perfect knowledge of the random variable's distribution?** This talk assumed there was no uncertainty in the RV. This makes a meta question, as the RV itself has a distribution now. Regret based approaches robustness... also related to learning / MAB literature (Gittin's, UCB, Boltzmann) dynamic experiments (clinical trials) and pricing.
- **Non-static distribution or distribution of distributions?** Can collapse down – just a higher dimensional policy? State / Markov decision processes.
- **Descriptive approaches in humans** – Prospect theory in humans to explain biases like risk aversion (or seeking in the case of gambling) Kahneman received the 2002 Nobel Prize in economics for this, Allais paradox, some non-transitivity from poor cognition? 1944 Von Neumann and Morgenstern proved rational preferences, by some reasonable definition, must use expected utility.
- **Effects of significant tail events / extinction risk (or finite time) / finite population effects** e.g. CVaR[X]

Jim Simons: mathematician made multi-millionaire

- Mathematician who founded Renaissance Technologies, as of passing had about 30 billion dollars.
- Founded the Simons foundation, major US funder of math research.



References

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Extras

- To the right we see the dice payoff graph for all ways of putting 6 eggs into three baskets, sorted clockwise from the top by outdegree (~overall strength). Indeed we see $[0,0,6]$ the all in one is worst, and $[2,2,2]$ is generally the best.

Dominance Graph: Non-redundant Strategies for $n=6$, $k=3$, Uniform dice_weighted_max

